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B.Sc. Part II (Hons.) 4^{th} Paper

DYNAMICS

Q. A particle moves along a circle $r = 2a\cos\theta$ in such a way that its acceleration towards the origin is always zero. Prove that $\frac{d\omega}{dt} = -2\omega^2\cot\theta$, where $\omega = \frac{d\theta}{dt}$.

Soln Given that

acceleration towards origin $= 0$

ie. radial acceleration $= 0$.

$$\Rightarrow \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = 0 \qquad \text{But } \omega = \frac{d\theta}{dt}$$

$$\Rightarrow \frac{d^2r}{dt^2} - r\omega^2 = 0, \Rightarrow \frac{d^2r}{dt^2} = r\omega^2. \quad -(1)$$

Also, given $r = 2a\cos\theta$

$$\Rightarrow \frac{dr}{dt} = -2a\sin\theta\frac{d\theta}{dt}$$

$$\Rightarrow \frac{d^2r}{dt^2} = -2a\cos\theta\left(\frac{d\theta}{dt}\right)^2 - 2a\sin\theta.\frac{d^2\theta}{dt^2} \quad -(2)$$

But $\frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$

So, (2) $\Rightarrow \frac{d^2r}{dt^2} = -2a\cos\theta.\left(\frac{d\theta}{dt}\right)^2 - 2a\sin\theta.\frac{d\omega}{dt}$

$$\Rightarrow r\omega^2 = -2a\cos\theta.\omega^2 - 2a\sin\theta.\frac{d\omega}{dt}$$

$$\Rightarrow r\omega^2 = -r\omega^2 - 2a\sin\theta\frac{d\omega}{dt} \quad [r = 2a\cos\theta]$$

$$\Rightarrow 2a\sin\theta\frac{d\omega}{dt} = -2r\omega^2 \quad \cancel{\Rightarrow \frac{d\omega}{dt} =}$$

$$\therefore \frac{d\omega}{dt} = \frac{-2r\omega^2}{2a\sin\theta} = \frac{-2 \cdot 2a\cos\theta \cdot \omega^2}{2a\sin\theta}$$

$$\Rightarrow \frac{d\omega}{dt} = -2\omega^2 \cot\theta.$$

Proved

Q A particle describes the curve $r = ae^{\theta}$ with constant angular velocity. Show that its radial acceleration is zero and the transverse acceleration varies as its distance from the pole.

Soln Given that angular velocity = constant

$$\Rightarrow \frac{d\theta}{dt} = k \text{ (suppose)} \quad \text{—(1)}$$

$$\because r = ae^{\theta} \text{ —(2)} \Rightarrow \frac{dr}{dt} = ae^{\theta} \cdot \frac{d\theta}{dt} = ae^{\theta} \cdot k \text{ [using (1)]}$$

$$\Rightarrow \frac{dr}{dt} = ae^{\theta} \cdot k = rk \text{ [using (2)]} \quad \text{—(3)}$$

$$\Rightarrow \frac{d^2r}{dt^2} = k \cdot \frac{dr}{dt} = k \cdot rk \text{ [using (3)]} = rk^2 \quad \text{—(4)}$$

Now, radial acceleration $= \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$

$$= rk^2 - r \cdot k^2 \text{ [using (4) and (1)]}$$

$$= 0$$

Proved

Transverse acceleration

$$= \frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = \frac{1}{r}\frac{d}{dt}(r^2 \cdot k) \text{ [using (1)]}$$

$$= \frac{1}{r}\frac{d}{dt}(kr^2) = \frac{1}{r} \cdot k\frac{d}{dt}(r^2) = \frac{k}{\cancel{r}} \times 2\cancel{r}\frac{dr}{dt}$$

$$= 2k\frac{dr}{dt} = 2k \times rk \text{ [from (3)]} = 2k^2 r \Rightarrow \text{Tr. acc}^n \propto r$$